Input design for controlling dynamics in a second-order memristive circuit

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Preliminaries

- Circuits featuring **Memristors** are characterized by high degrees of **multistability**[1].
- Multistability can be exploited to develop **new Computational Paradigms** (e.g., in the reservoir computing framework[2]).
- The state space of circuits with **Ideal Memristors** is **foliated** into a continuum of **Invariant Manifolds**.
- **Feed-forward[3] and Feedback[4] Control Laws** can be used to target the circuit dynamics towards an attractor contained in one of the invariant manifolds.

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[1] Bao B. et al. Coexisting infinitely many attractors in active band-pass filter-based memristive circuit. Nonlinear Dynamics, 2016, 86.3: 1711-1723.

[2] Traversa F. L. and Di Ventra M. Universal memcomputing machines. IEEE transactions on neural networks and learning systems, 2015, 26.11: 2702-2715.

[3] Corinto F. and Forti M. Memristor circuits: Pulse programming via invariant manifolds. IEEE Transactions on Circuits and Systems I: Regular Papers, 2017, 65.4: 1327-1339.

[4] Di Marco M. et al. Control Design for Targeting Dynamics of Memristor Murali-Lakshmanan-Chua Circuit. In: 2019 18th European Control Conference (ECC). IEEE, 2019. p. 4332-4337.

The illustrative circuit

Aim of the work

Introducing a procedure to **design control inputs** able to target a **desired behavior**, i.e.

- to drive within a prescribed time interval the dynamics from one manifold to another (switching among different attractors),
- to modify the dynamics onto each invariant manifold.

The procedure will be illustrated by designing the **controlled sources** of the RLC circuit in Fig. 1.

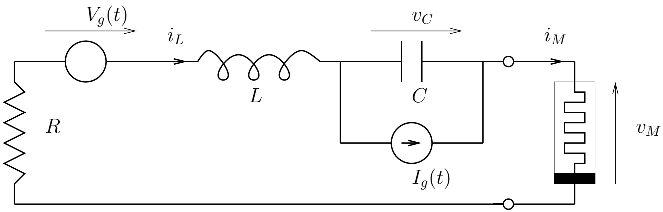

Figure 1: The circuit features a (locally active) charge-controlled memristor connected to a passive RLC two-terminal element, where the current generator I_g and the voltage generator V_g represents the controlled sources object of the design. The input-less (or uncontrolled) circuit is obtained for $I_g = V_g = 0$.

The circuit dynamics

The **one-port circuit** of Fig. 1, composed by a resistor R, an inductor L, a capacitor C, a current source I_g and a voltage source V_g, is a linear two-terminal element governed by the differential equations

$$\begin{cases} \mathcal{D}v_C(t) = \dfrac{1}{C}\left(I_g(t) - i_L(t)\right) \\ \mathcal{D}i_L(t) = \dfrac{1}{L}\left(V_g(t) + v_C(t) - Ri_L(t) - v_M(t)\right) \end{cases} \tag{1}$$

where

- $\mathcal{D}$ is the time-derivative operator,
- v_C is the capacitor voltage,
- i_L is the inductor current,
- v_M is the voltage of the charge-controlled memristor.

The circuit dynamics

The **memristor** is described by the following model

$$\begin{cases} \mathcal{D}q_M(t) = i_M(t) \\ v_M(t) = M(q_M(t))i_M(t) \ , \end{cases} \tag{2}$$

where

- i_M and q_M are the memristor current and charge,
- M is the **memristance**, defined as the derivative[5] of the **charge-flux characteristic** $\hat{\varphi} : \mathbb{R} \to \mathbb{R}$, i.e.,

$$M(q_M) = \hat{\varphi}'(q_M) \ .$$

Charge-flux characteristic

In this work it is assumed

$$\hat{\varphi}(q_M) = -\alpha_0 q_M + \frac{\alpha_1}{3} q_M^3 \ , \quad \alpha_0 > R, \ \alpha_1 > 0. \tag{3}$$

[5]Symbol $'$ stands for derivative against the function argument.

The circuit dynamics

The **complete dynamics** is governed by the following equation system:

$$\Sigma : \begin{cases} \mathcal{D}v_C(t) = & \frac{1}{C}\left(I_g(t) - i_L(t)\right) \\ \mathcal{D}i_L(t) = & \frac{1}{L}\left(V_g(t) + v_C(t) - Ri_L(t) - \hat{\varphi}'(q_M(t))i_L(t)\right) \\ \mathcal{D}q_M(t) = & i_L(t) \ . \end{cases} \tag{4}$$

The next result shows that

- the dynamics of the uncontrolled circuit can be seen as a collection of the dynamics of a **family of reduced order systems**, denoted as $\Sigma_{\mathcal{M}(Q_0)}$,
- the family of reduced order systems can be parameterized by means of the **index** $Q_0 = Q(t_0)$ which depends from the circuit initial conditions according to its definition

$$Q(t) \doteq q_M(t) + Cv_C(t) \ . \tag{5}$$

Invariant manifolds

Proposition 1

Let $I_g(t) = V_g(t) = 0$ for all $t \geq t_0$, i.e., the circuit of Fig. 1 is uncontrolled. Then, the state space of Σ is composed by **infinitely many invariant manifolds** of the form

$$\mathcal{M}(Q_0) = \left\{(v_C, q_M, i_L) : \; q_M(t) + Cv_C(t) = Q_0, \; \forall t \geq t_0\right\},$$

where $Q_0 \in \mathbb{R}$ is the **manifold index**. Moreover, the second-order system of differential equations

$$\Sigma_{\mathcal{M}(Q_0)} : \begin{cases} \mathcal{D}q_M(t) & = i_L(t) \\ \mathcal{D}i_L(t) & = \dfrac{1}{LC}(Q_0 - q_M(t)) - \dfrac{R}{L} i_L(t) - \dfrac{1}{L}\hat{\varphi}'(q_M(t)) i_L(t) \end{cases} \tag{6}$$

describes the dynamics onto $\mathcal{M}(Q_0)$.

Invariant manifold dynamics

The next result characterizes the equilibrium points of $\Sigma_{\mathcal{M}(Q_0)}$ and their stability properties.

Proposition 2

$\Sigma_{\mathcal{M}(Q_0)}$ has a **unique equilibrium point** at $(q_M, i_L) = (Q_0, 0)$ which is locally asymptotically **stable** if $|Q_0| > \bar{Q}_0$ and **unstable** if $|Q_0| < \bar{Q}_0$, where

$$\bar{Q}_0 = \sqrt{\frac{\alpha_0 - R}{\alpha_1}} \ .$$

Remark 1

At $Q_0 = \pm\bar{Q}_0$ a **Hopf bifurcation** is likely to occur. Since the bifurcation is generated by varying the index Q_0 for fixed values of circuit parameters R, L, C, α_0, α_1, it may be referred to as a **"bifurcation without parameters"** [a].

[a] Fiedler B. et al. Generic Hopf bifurcation from lines of equilibria without parameters: I. Theory. Journal of Differential equations, 2000, 167.1: 16-35.

Steering the dynamics from one invariant manifold to another

Problem 1

Assuming that for $t \in [t_0, t_1]$, $t_1 > t_0$, the dynamics of the (uncontrolled) circuit of Fig. 1 lies onto the invariant manifold $\mathcal{M}(Q_0^{(1)})$, drive the circuit towards $\mathcal{M}(Q_0^{(2)})$, $Q_0^{(2)} \neq Q_0^{(1)}$, within a given time interval $t \in (t_1, t_2)$, $t_2 > t_1$, and reaching it at $t = t_2$.

Lemma 1

Along the solutions of (4) we have the relation

$$\mathcal{D}Q(t) = I_g(t) \quad \forall t \geq t_0 \ . \tag{7}$$

Lemma 1 clarifies that the voltage source V_g does not yield changes in $Q(t)$, i.e., $Q(t) = Q_0$ for all $t \geq t_0$. This means that V_g **cannot be used for switching** from an invariant manifold to another one.

Steering the dynamics from one invariant manifold to another

Proposition 3

Problem 1 can be solved by letting $V_g(t) = 0$ for all $t \geq t_0$ and

$$I_g(t) = \begin{cases} 0 & \text{if} \quad t \in [t_0, t_1] \\ w(t) & \text{if} \quad t \in (t_1, t_2) \\ 0 & \text{if} \quad t \in [t_2, +\infty) \end{cases} \tag{8}$$

where $w(t)$ is any piece-wise continuous function such that:

$$Q_0^{(2)} - Q_0^{(1)} = \int_{t_1}^{t_2} w(t)dt \ . \tag{9}$$

Steering the dynamics from one invariant manifold to another

Corollary

The circuit dynamics lies onto $\mathcal{M}(Q_0^{(1)})$ for $t \in [t_0, t_1]$ and then reaches $\mathcal{M}(Q_0^{(2)})$ at $t = t_2$, where it remains for $t \geq t_2$. Moreover, in the time interval $[t_1, t_2]$ the behavior of the state variables i_L and q_M of Σ is completely described by the second and the third equations of (4) with $V_g(t) = 0$ and

$$v_C(t) = \frac{1}{C}\left(Q_0^{(1)} + \int_{t_1}^{t} w(\tau)d\tau - q_M(t)\right) \ . \tag{10}$$

Remark 2

All the pulse inputs $w(t)$ satisfying the **area constraint** (9) are able to steer the circuit dynamics from $\mathcal{M}(Q_0^{(1)})$ to $\mathcal{M}(Q_0^{(2)})$ within the time interval $[t_1, t_2]$.

Modifying the dynamics onto an invariant manifold

Since V_g cannot be used for switching from an invariant manifold to another one, it is worth investigating if it can be exploited to alter the dynamics onto the invariant manifolds.

Problem 2

Assuming $I_g(t) = 0$ for all $t \geq t_0$, verify if the attractors on the invariant manifolds of the uncontrolled circuit can be modified using the **feedback control law**

$$V_g(t) = \gamma i_L(t) \ , \tag{11}$$

where the **constant gain** γ is the object of the designed.

Note that (11) is a **linear time-invariant state feedback** which only requires to measure the **inductor current** i_L.

Modifying the dynamics onto an invariant manifold

Proposition 4

Let $I_g(t) = 0$ for all $t \geq t_0$, and let $V_g(t)$ be as in (11). Then, each $\mathcal{M}(Q_0)$ has a **unique equilibrium point** at $(q_M, i_L) = (Q_0, 0)$ which is locally asymptotically **stable** if $|Q_0| > \bar{Q}_0(\gamma)$ and **unstable** if $|Q_0| < \bar{Q}_0(\gamma)$ where

$$\bar{Q}_0(\gamma) = \sqrt{\frac{\alpha_0 - \gamma - R}{\alpha_1}}\ . \tag{12}$$

Proposition 4 proves that **feedback control law** (11)

- does not alter the foliation property of the state space,
- is able to modify the stability of the fixed points on the manifolds $\mathcal{M}(Q_0)$.

Numerical simulations

The previous results are verified via **numerical simulations** assuming the following parameters:

$$R = 0.4, \quad C = 0.1, \quad L = 1.5, \quad \alpha_0 = 0.7, \quad \alpha_1 = 0.3, \tag{13}$$

which imply $\bar{Q}_0 = 1$.

The above configuration of the **uncontrolled circuit** features on each manifold

- a stable equilibrium point if $|Q_0| > 1$,
- an unstable equilibrium point surrounded by a stable limit cylce if $|Q_0| < 1$.

Figure 2 provides a graphical overview of the attractors in the circuit state space.

Numerical simulations

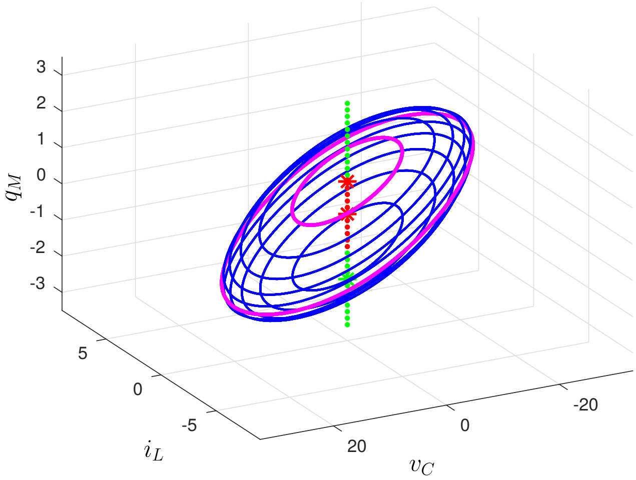

Figure 2: Equilibrium points of the uncontrolled circuit, stable (green) and unstable (red). Stable limit cycles generated by the (supercritical) Hopf bifurcations "without parameters" are denoted in blue. Stationary solutions for $Q_0 = -1.8$ (a stable equilibrium), $Q_0 = 0$ (an unstable equilibrium and a stable limit cycle), and $Q_0 = 0.9$ (an unstable equilibrium and a stable limit cycle) have been highlighted with an asterisk (equilibrium points) and in magenta (limit cycles).

Numerical simulations

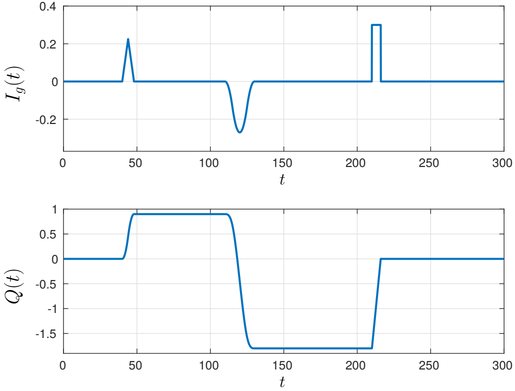

Figure 3: Upper plot - Differently shaped current pulses are applied as control signal $I_g(t)$. Lower plot - Evolution of index $Q(t)$ due to control signal $I_g(t)$ among $Q_0 = 0$, $Q_0 = 0.9$, $Q_0 = -1.8$, and final return to $Q_0 = 0$, i.e., the total area of the pulses is null.

Numerical simulations

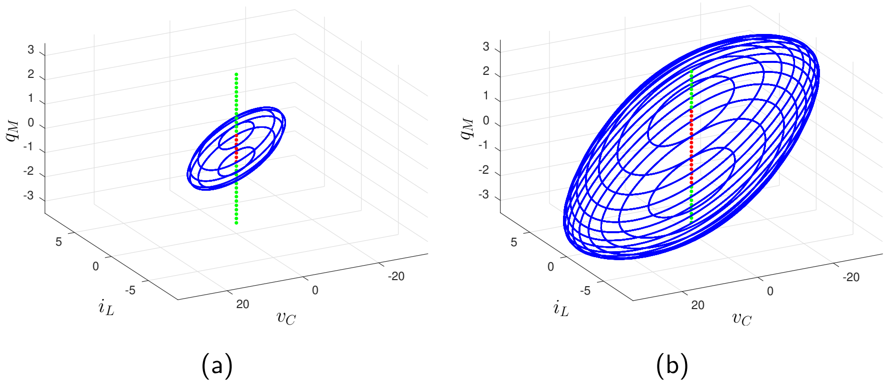

Figure 4: According to Proposition 4, γ moves the Hopf bifurcation point, thus modifying the ranges of stable equilibria and stable limit cycles. a) Equilibrium points of system Σ: stable (green), unstable (red) and limit cycles for $\gamma = 0.2$. b) Equilibrium points of system Σ: stable (green), unstable (red) and limit cycles for $\gamma = -0.4$.

Conclusions

- The problem of controlling the dynamics of a circuit composed by a two-terminal element containing RLC components and a (locally active) **memristor** has been addressed.
- Control design has taken into account both **steering the circuit dynamics** from one invariant manifold to another (switching among different attractors) and **modifying the dynamics** onto each invariant manifold.
- Both the above issues can be addressed using **controlled sources** placed in the two-terminal element: the first issue is solved by **impulsive feedforward control inputs**, the second by a **linear feedback control law** based only on the **inductor current**.

THANKS FOR YOUR ATTENTION!