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Spatial Context Tree Weighting for Physical Unclonable Functions

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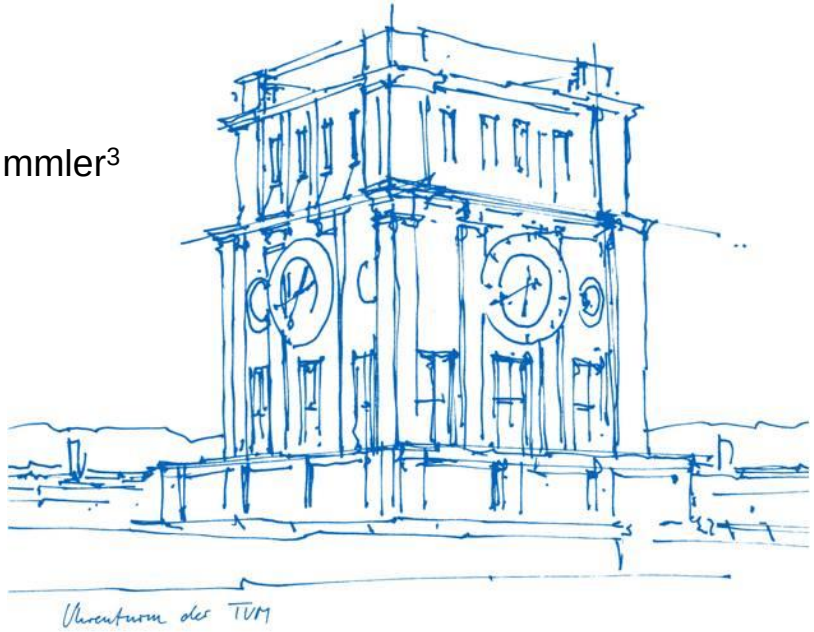


Agenda

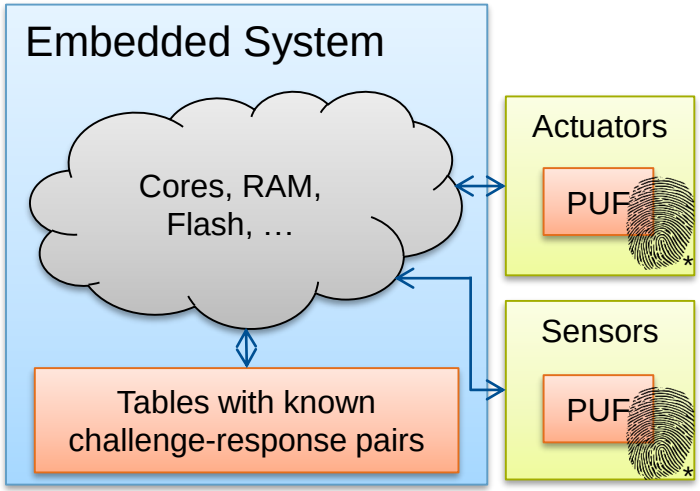
- Entropy Estimation of Physical Unclonable Functions:

  Motivation and State of the Art

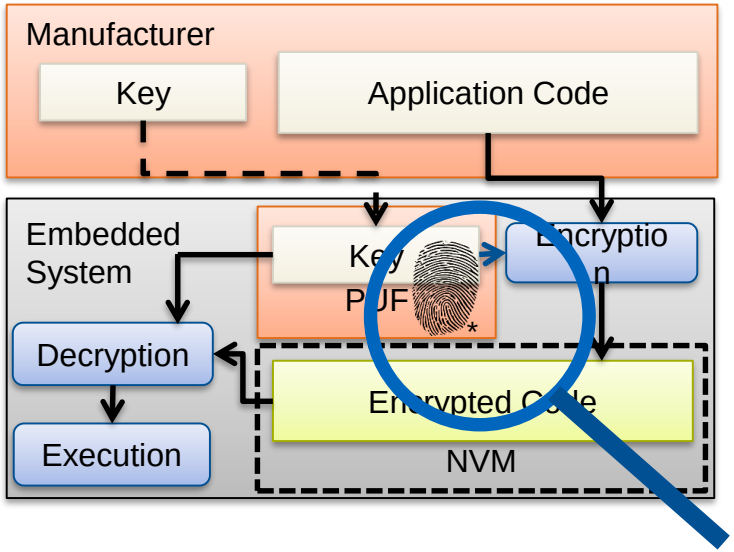
- Spatial Context Tree Weighting
- Practical Results of Entropy Estimation
- Conclusion

Physical Unclonable Functions (PUFs)

PUF for Authentication/Identification



PUF for Key-Storage



- PUFs provide a device unique secret
- The security level strongly depends on the PUF‘s quality

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State of the Art in Weak PUF Evaluation

Statistical Metrics:

- Maiti et al., *A Systematic Method to Evaluate and Compare the Performance of Physical Unclonable Functions:* Bit-Alias, Uniformity, Uniqueness …
- Wilde et al., *On the Confidence in Bit-Alias Measurement of Physical Unclonable Functions:* Confidence intervals on Bit-Alias.
- Hori et al., *Quantitative and Statistical Performance Evaluation of Arbiter Physical Unclonable Functions on FPGAs*: Randomness, Diffuseness, Uniqueness …
- Wilde et al., *Statistic-based security analysis of ring oscillator PUFs:* PCA.
- Wilde et al., *Spatial Correlation Analysis on Physical Unclonable Functions:* Statistical analysis of spatial effects.

→ Provide various insides in PUFs, but hard to interpret.

State of the Art in Weak PUF Evaluation

How hard is it to guess the key? → Entropy Metrics:

- Ignatenko et al., *Estimating the Secrecy-Rate of Physical Unclonable Functions with the Context-Tree Weighting Method:* First to apply context tree weighting to PUFs, spatial dependencies not systematically analyzed.
- Pehl et al., *Efficient Evaluation of Physical Unclonable Functions Using Entropy Measures:* Dependency of entropy and potential defects analyzed, no spatial dependencies considered.
- Wilde et al., *Efficient Bound for Conditional Min-Entropy of Physical Unclonable Functions Beyond IID:* Estimation of entropy remaining in a key, no spatial dependencies considered

→ Our goal: Consider spatial dependencies to improve entropy bounds for array like PUFs

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Context Tree Weighting (CTW)*

Given: Sequence of symbols $x = x_0, x_1, \dots, x_N$

Assumption: current symbol depends only on last D symbols

Algorithm:

- Model source as a tree source with depth D
- Approximate probability of sequences to appear
- Probability of the root node (coding probability P_c) is a measure for entropy

Entropy estimation for PUFs:

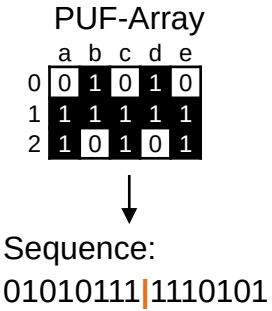
- Entropy in the sequence (PUF): $H_{PUF} \leq -\frac{\log_2 P_c}{N-D}$

* Willems et al. *Context Tree Weighting : A Sequential Universal Source Coding Procedure for FSMX Sources*

Context Tree Weighting (CTW)*

Given: Sequence of symbols $x = x_0 x_1, \dots, x_N$

Assumption: current symbol depends only on last D symbols → **Let $D = 8$**



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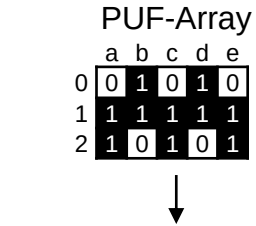

Context	Following Zeros/Ones
01010111	0/1

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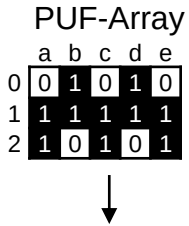

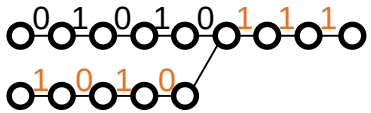

Context	Following Zeros/Ones
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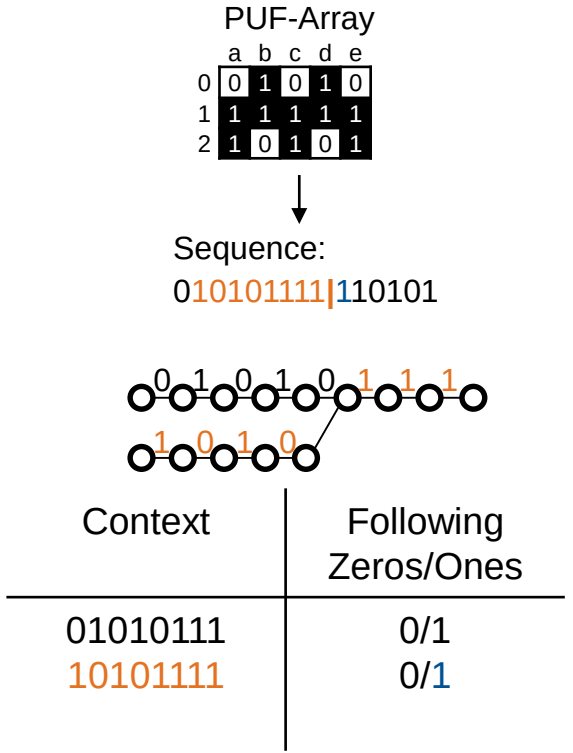
Entropy Estimation of PUFs with CTW

- CTW provides an upper bound for entropy
- No spatial ordering considered
  → sequences w/o physical meaning used for entropy estimation
- Dependencies are considered within the last D elements
  → likely dependent responses might be not in the same sequence



From CTW to Spatial Context Tree Weighting (SCTW)

Classical CTW (CCTW)

- Considers only dependencies in last D elements
  → *Relevant* PUF responses shall appear in the context

PUFs

- Are frequently arranged in arrays
- Are rather correlated when spatially close
  → Responses of spatially close PUFs are *relevant*.

⇒ Re-Interpretation of context:
CCTW: Context of conditioned symbol = last symbols in sequence.
SCTW: Context of conditioned symbol = neighboring bits.
We have proven: SCTW still approximates entropy

CCTW-Context

	a	b	c	d	e
0	0	1	0	1	0
1	1	1	1	1	1
2	1	0	1	0	1

SCTW-Context

	a	b	c	d	e
0	0	1	0	1	0
1	1	1	1	1	1
2	1	0	1	0	1

Considered CTW-Variants

- All variants: consideration of higher order alphabet responses
- Classical CTW (CCTW):
  - Responses concatenated row-wise
- Spatial CTW (SCTW):
  - Considers neighboring bits
  - Well suited to detect spatial defects
- Forward CTW (FCTW)
  - Similar to SCTW
  - Less balanced: some bits only seldom in context
- Horizontal CTW (HCTW) / Vertical CTW (VCTW)
  - Well suited to find correlations in rows/columns
  - Help to identify specific defects

	a	b	c	d	e	f	g	h	i	j	k	l	m	n	o
0										7			8	7	6
1	5	4	3	2	1	X				5					
2										3					
3		8	1	5						1					
4		4	X	2						X		X	1	3	
5		7	3	6						2		2	4	6	
6										4		5	7	8	
7	7	5	3	1	X	2	4	6	8	6					
8										8					

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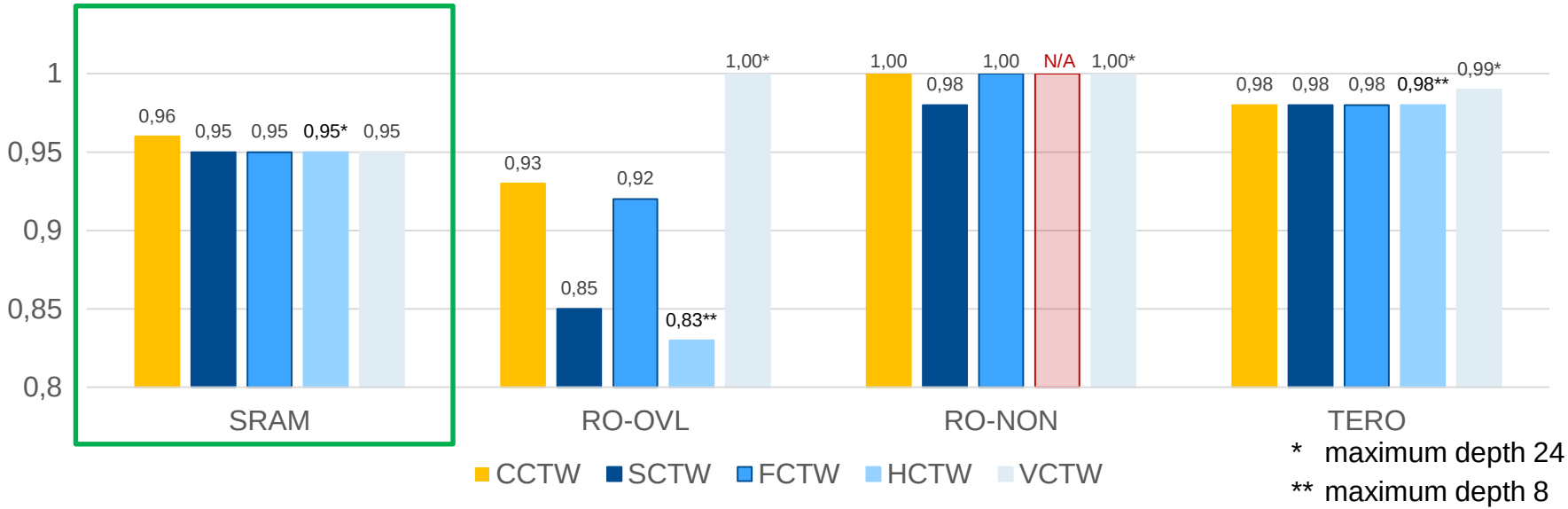
Analyzed Data

5 different PUF designs:

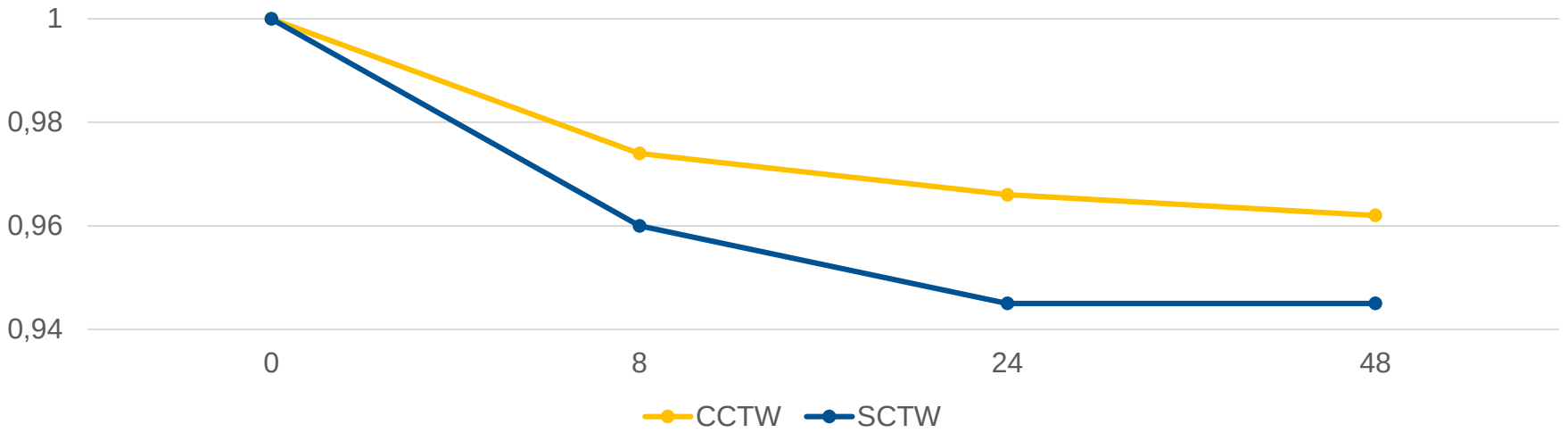
- SRAM (cf. Wilde, *Large Scale Characterization of SRAM on Infineon XMC Microcontrollers As PUF*):
  - Arrays of 32X40960 SRAM cells from 144 Microcontrollers (actual layout unknown)
- RO-OVL (cf. Maiti et al., *A Large Scale Characterization of RO-PUF*):
  - Arrays of 32X15 response bits derived from arrays of 32X16 ROs on 193 FPGAs
- RO-NON (cf. Maiti et al., *A Large Scale Characterization of RO-PUF*):
  - Arrays of 32X8 response bits derived from arrays of 32X16 ROs on 193 FPGAs
- TERO (cf. Wild et al., *A fair and comprehensive large-scale analysis of oscillation-based PUFs for FPGA*):
  - Arrays of 16X40 response bits derived from 16X80 TERO cells on 100 FPGAs
- Tamper-Evident PUF (cf. Immler et al. Secure Physical Enclosures from Covers with Tamper-Resistance):
  - Arrays of 8X16 capacitances each providing a 5-bit response symbol

Estimated Entropy for Different Weak PUFs (Tree Depth 48)



SCTW improves estimate in all experiments

Entropy over Tree Depth: CCTW vs SCTW



SCTW covers important dependencies early

→ Better approximation with lower tree depth possible

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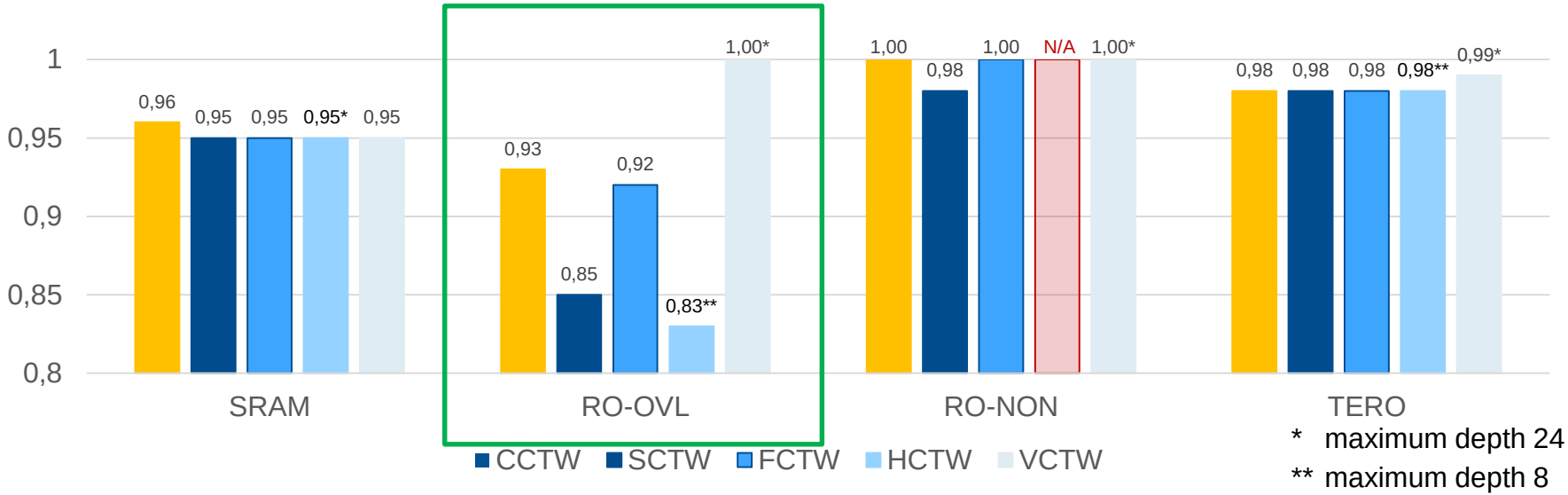

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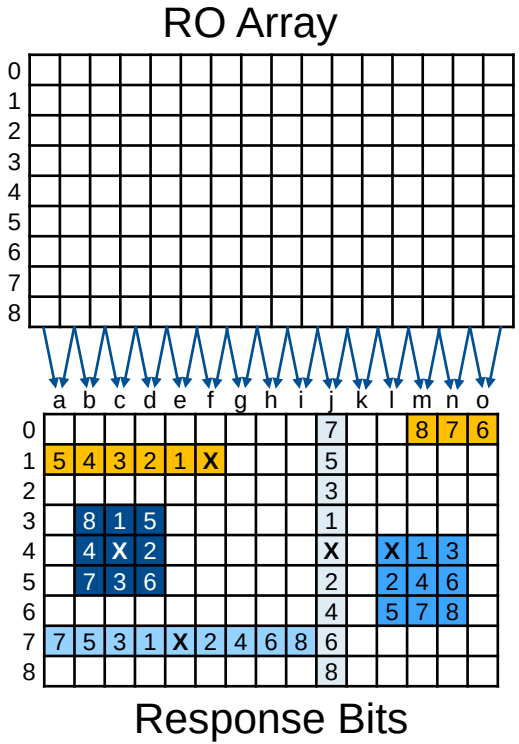
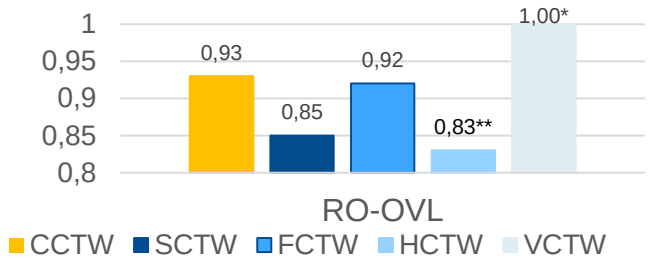
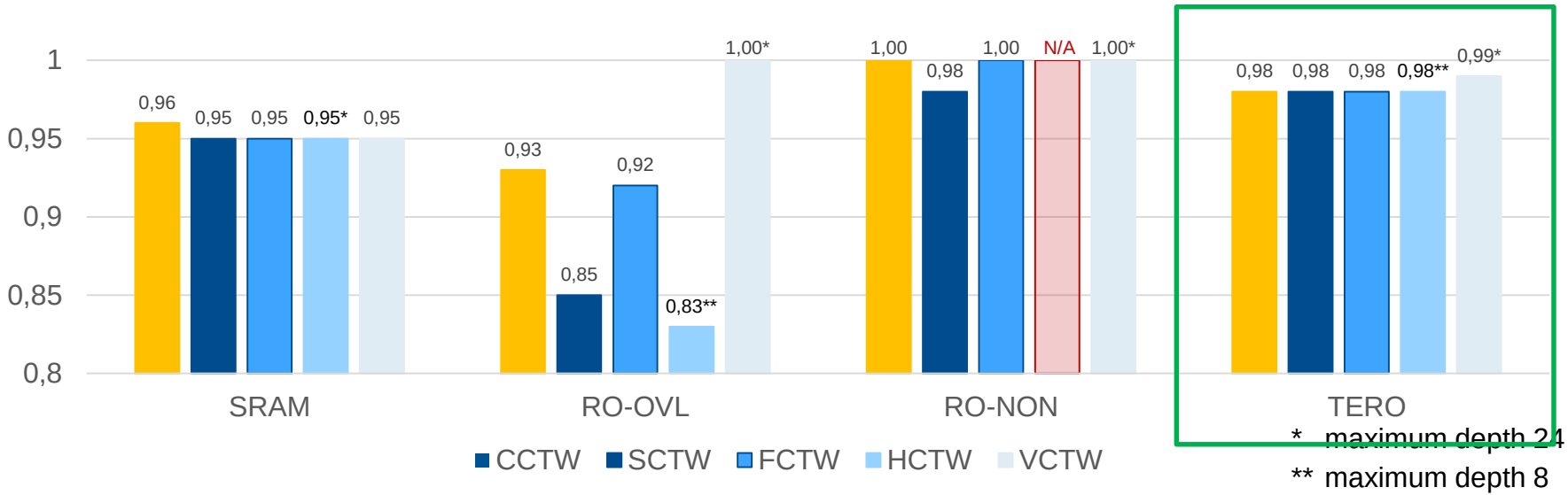




Frequencies compared in overlapping manner

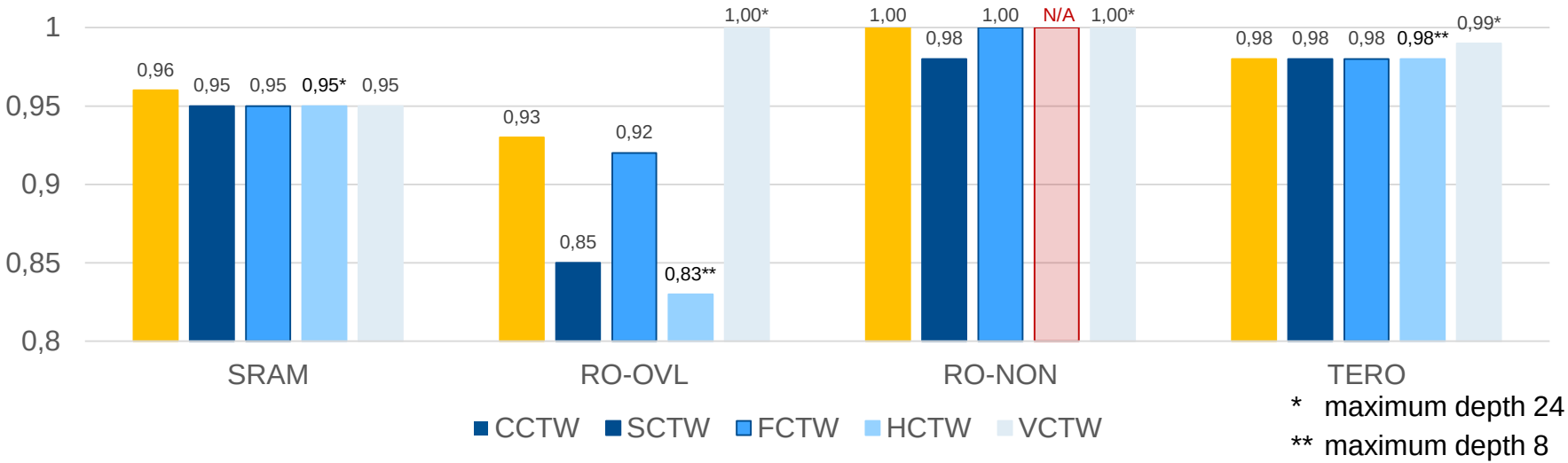
- → Correlation of bits in lines
- → CCTW: Estimate over line breaks
- → SCTW: Spatial dependencies in all directions
- → FCTW: Similar to SCTW but less well balanced
- → HCTW: Considers here most of the relevant dependencies with smallest depth
- → VCTW: Does not consider relevant dependencies

Estimated Entropy for Different Weak PUFs (Tree Depth 48)



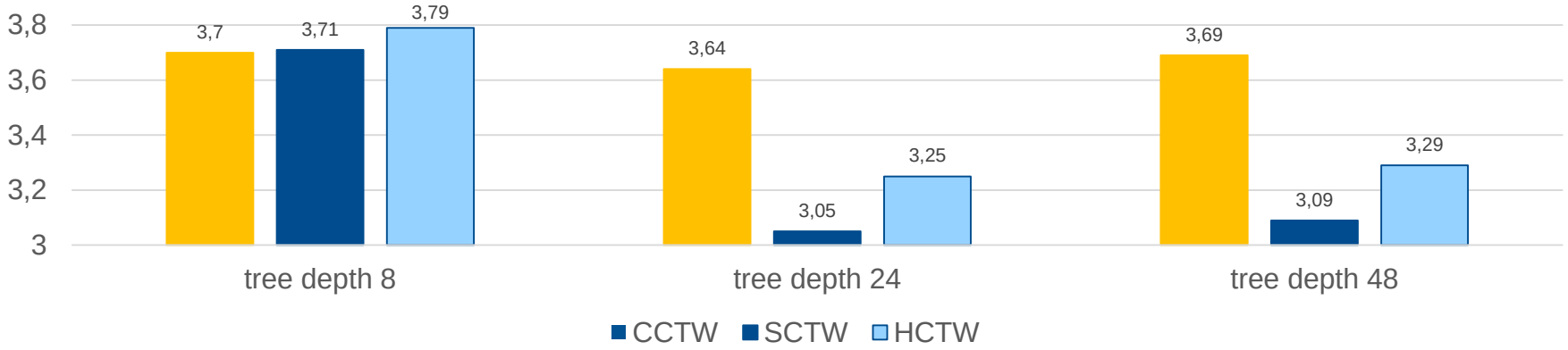
CCTW and SCTW result in same entropy → likely pure bias and no spatial defects

Estimated Entropy for Different Weak PUFs (Tree Depth 48)



- In general CCTW provides best estimate w/o pre knowledge
- Other context shapes: insights to possible design problems

Higher Order Alphabet PUF



- Target of analysis: tamper-evident PUF with 5 bit responses per position
- Significant improve of SCTW over CCTW
- Slide increase from dree depth 24 to 48 caused by numeric

Conclusion

We provided

- Extension of CTW to SCTW and further spatial variants

Benefits:

- Improved entropy bound through consideration of spatial effects.
- Application of different spatial variants reveals possible design issues.

Results:

- Analysis of 5 different PUF implementations
- Improved estimate of entropy (up to 16% for higher-order alphabet PUF)
- Improved bound given same tree depth (i.e., same computational effort)

Contact and Acknowledgement

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