24th European Conference on Circuit Theory and Design
September 7-10, 2020, Sophia, Bulgaria

A High Speed and Low Complexity Architecture Design Methodology for Square Root Unscented Kalman Filter based SLAM

RASHI DUTT
Dept. of Electrical Engineering
IIT Hyderabad
ee16resch11014@iith.ac.in

AMIT ACHARYYA
Dept. of Electrical Engineering
IIT Hyderabad
amit_acharyya@ee.iith.ac.in

भारतीय प्रौद्योगिकी संस्थान हैदराबाद
Indian Institute of Technology Hyderabad

भारतीय प्रौद्योगिकी संस्थान हैदराबाद
Indian Institute of Technology Hyderabad

PLAN OF THE TALK

भारतीय प्रौद्योगिकी संस्थान हैदराबाद
Indian Institute of Technology Hyderabad

ABSTRACT

- Square Root Unscented Kalman Filter (SRUKF) provides a practical solution for highly nonlinear and critical applications such as Simultaneous Localization and Mapping (SLAM).
- SRUKF improves the stability of the system, enhancing the numerical accuracy for hardware platform.
- SRUKF is less computationally demanding than Unscented Kalman Filter (UKF).
- Hardware development process remains a complex task with high resource utilization.
- Householder CORDIC based low complexity SRUKF architecture for SLAM application is proposed.
- A hardware-software co-design methodology is proposed and implemented on Zynq-7000 XC7Z020 FPGA.
- Synthesis results show the architecture saves 91% of DSP cores and is 20% faster than UKF.
- The proposed SRUKF is highly stable and achieve 78% and 10% greater accuracy than EKF SLAM and UKF SLAM respectively.

***Index Terms*— Square Root Unscented Kalman Filter, Simultaneous Localization and Mapping, Low Complexity Architecture, Householder CORDIC**

भारतीय प्रौद्योगिकी संस्थान हैदराबाद
Indian Institute of Technology Hyderabad

INTRODUCTION

- Kalman Filter and its variants form a class of robust and optimal state estimators.
- Critical applications of Kalman Filter include - **vehicle guidance, satellite trajectory control and most recently in the field of autonomous navigation.**
- SLAM enables
  - Estimating the position of an autonomously moving robot
  - Building a map of its environment
  - Both the robot position and the true map of the environment are unknown
- Extended Kalman Filter (EKF) is a widely used algorithm for developing SLAM.
- EKF linearizes the nonlinear system that limits its accuracy upto first order.
- For the highly nonlinear SLAM, **EKF fails to converge**.

भारतीय प्रौद्योगिकी संस्थान हैदराबाद
Indian Institute of Technology Hyderabad

LITERATURE REVIEW

- Several new variants of Kalman Filter other than EKF proposed in the literature, such as
  - Unscented Kalman Filter (UKF)
  - FastSLAM
- UKF overcomes linearization by modeling the nonlinear system as a stochastic system.
- UKF is less computationally intensive compared to EKF - no Jacobians are computed.
- UKF requires fewer number of deterministic sigma points - useful for computationally intensive SLAM.
- Critical point of failure in UKF making the system fatally unstable –
  - **Error covariance matrix becomes progressively ill-conditioned**
  - **Error covariance matrix loses its positive semi-definite nature**
- Square root Unscented Kalman Filter propagates the square root of the error covariance matrix.
- SRUKF based SLAM is more stable than UKF and also has higher numerical accuracy.
- **Literature highlights the need for improved architecture design for real-time SLAM implementations.**

भारतीय प्रौद्योगिकी संस्थान हैदराबाद
Indian Institute of Technology Hyderabad

MAIN CONTRIBUTIONS

- SRUKF based SLAM algorithm is formulated and for the first time its low-complexity architecture design is proposed.
- No hardware architecture design for SRUKF is present till date to the best of the authors' knowledge.
- A high speed and low-complexity Architecture Design Methodology is proposed.
- Low complexity Householder CORDIC based triangularisation and cholesky update/downdate processors are developed.
- A precomputation strategy is proposed to store the square root of the error covariance matrix to reduces the computational complexity.
- Hardware software co-design methodology is proposed for ease of portability.

भारतीय प्रौद्योगिकी संस्थान हैदराबाद
Indian Institute of Technology Hyderabad

BACKGROUND

- For SLAM, the augmented state vector comprises
  - Robot state - described by its position and orientation
  - Observational position of the landmarks
- SRUKF SLAM provides a probabilistic approach for representing the motion and observation models.
- Localization algorithm is initialized by mean and square root of the state covariance matrix.
- A set of sigma points are transformed through the nonlinear process model.
- Cross-covariance between the state and observation vector is computed for calculating Kalman Gain.
- Finally, the updated state and square root of error covariance matrix are computed using Kalman gain.
- A new landmark is observed and its position is predicted for the next iteration.
- Updated state and covariance matrix is augmented with mean and square root of error covariance matrix for a new landmark.

Proposed SRUKF Algorithm for SLAM

Algorithm 1 Proposed SRUKF SLAM

SRUKF Initialization
Mean: $\hat{x}_0 = E[x_0]$
Error Covariance: $S_{k,0} = chol\{E[(x_0 - \hat{x}_0)(x_0 - \hat{x}_0)^T]$
Localization
Sigma Point Computation:
$\mathcal{X}^-_{k-1} = [\hat{x}_{k-1} \quad \hat{x}_{k-1} + \eta S_{k-1} \quad \hat{x}_{k-1} - \eta S_{k-1}]$
Time Update:
$\mathcal{X}^*_{k|k-1} = f(\mathcal{X}^-_{k-1}, \; u_{k-1})$
$\hat{x}_k^- = \sum_{i=0}^{2L} W_i^{(m)} \; \mathcal{X}^*_{i,k|k-1}$
$S_k^- = QRD\left\{\left[\sqrt{W_i^{(c)}}(\mathcal{X}^*_{1:2L,k|k-1} - \hat{x}_k^-) \quad \sqrt{R^n}\right]\right\}$
$S_k^- = cholupdate\{S_k^-, \; (\mathcal{X}^*_{i0,k} - \hat{x}_k^-), \; W_0^{(c)}\}$
Measurement Update:
$\mathcal{X}_{k|k-1} = [\hat{x}_k^- \quad \hat{x}_k^- + \eta S_k^- \quad \hat{x}_k^- - \eta S_k^-]$
$\mathcal{Y}_{k|k-1} = g(\mathcal{X}_{k|k-1}, \; u_k)$
$\hat{y}_k^- = \sum_{i=0}^{2L} W_i^{(m)} \; (\mathcal{Y}_{k|k-1})$
$S_{\hat{y}_k^-} = QRD\left\{\left[\sqrt{W_{1:2L}^{(c)}}(\mathcal{Y}_{1:2L,k} - \hat{y}_k^-) \quad \sqrt{R^v}\right]\right\}$
$S_{\hat{y}_k^-} = cholupdate\{S_{\hat{y}_k^-}, \; (\mathcal{Y}_{0,k} - \hat{y}_k^-), \; W_0^{(c)}\}$
Kalman Gain:
$P_{x_k y_k} = \sum_{i=0}^{2L} W_i^{(c)} [\mathcal{X}_{i,k|k-1} - \hat{x}_k^-] \; [\mathcal{Y}_{i,k|k-1} - \hat{y}_k^-]^T$
$\mathcal{K}_k = (P_{x_k y_k} / S^T_{\hat{y}_k^-}) / S_{\hat{y}_k^-}$
Updated State Error Covariance:
$\hat{x}_k = \hat{x}_k^- + \mathcal{K}_k(y_k - \hat{y}_k^-)$
$S_k = S_{\hat{y}_k^-} S^*_{\hat{y}_k^-} - \mathcal{K}_k S_{\hat{y}_k^-} S^*_{\hat{y}_k^-} \mathcal{K}_k^T$
New Landmark Initialization
$\mathcal{Z}^*_{k|k-1} = h(\mathcal{X}_{k|k-1}, \; L_k)$
$\hat{z}_{k,lm} = \sum_{i=0}^{2L} W_i^{(m)} \; \mathcal{Z}^*_{i,k|k-1}$
$S_{k,lm} = QRD\left\{\left[\sqrt{W_i^{(c)}}(\mathcal{Z}^*_{1:2L,k|k-1} - \hat{z}_{k,lm}) \quad \sqrt{R^l}\right]\right\}$
$S_{k,lm} = cholupdate\{S_{k,lm}, \; (\mathcal{Z}^*_{0,k} - \hat{z}_{k,lm}), \; W_0^{(c)}\}$

भारतीय प्रौद्योगिकी संस्थान हैदराबाद
Indian Institute of Technology Hyderabad

PROPOSED SRUKF ARCHITECTURE DESIGN METHODOLOGY

PROPOSED SRUKF ARCHITECTURE DESIGN METHODOLOGY (1/3)

- Hardware software co-design methodology is proposed for SRUKF SLAM.
- Application specific nonlinear process model and observation models developed and stored in the processor.
- Core computational unit consisting of vector multiplication and accumulation - implemented on FPGA.
- Proposed architecture design methodology enhances the performance in terms of :-
  - Reduced hardware complexity
  - Efficient power utilization
  - High speed computation

PROPOSED SRUKF ARCHITECTURE DESIGN METHODOLOGY (2/3)

Sigma point generation using pre-computed weights and Cholesky factorization

- Square root of covariance matrix for sigma points is required at the initialization step only.
- The square root form is subsequently propagated at each iteration.
- Cholesky factor is pre-computed and passed to the core computation unit.
- Weights associated with mean and covariance matrix are pre-computed.
- **Square root computation has been eliminated from the critical data-path of sigma point computation.**

Householder CORDIC based Triangularization

- A high speed Householder CORDIC based triangularization unit is developed.
- Householder CORDIC - combines the simple low complexity of CORDIC with high speed orthogonalization of Householder transformation.
- 4-Dimension QR Decomposition methodology is proposed using doubly pipelined CORDIC.
- The latency of the module is 8 clock cycles after the combined latency of doubly pipelined CORDIC.

भारतीय प्रौद्योगिकी संस्थान हैदराबाद
Indian Institute of Technology Hyderabad

PROPOSED SRUKF ARCHITECTURE DESIGN METHODOLOGY (3/3)

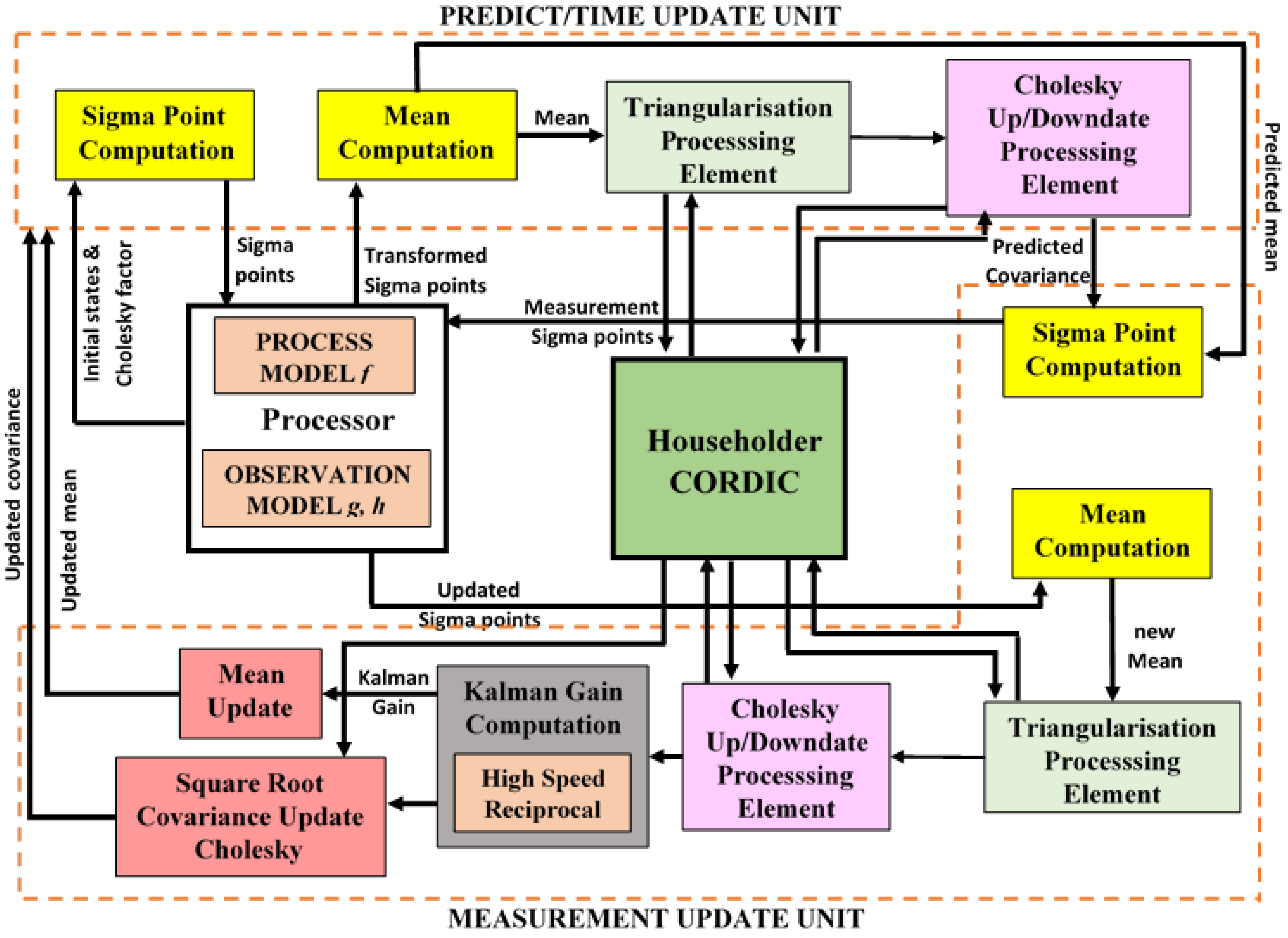
Cholesky Factor Update and Downdate

- For every landmark, the cholesky factor update and downdate is performed for three matrices.
- Householder based cholesky update/downdate processor is developed using 4D householder CORDIC.
- Householder CORDIC unit is used in a doubly pipelined manner.
- Householder CORDIC unit has been designed to include separate select lines to implement circular mode for cholesky update and hyperbolic mode for cholesky downdate.

Kalman Gain Computation

- SRUKF uses two least squares solutions for Kalman gain computation.
- The error covariance matrix is lower triangular matrix – back substitution is used.
- The back substitution requires a reciprocal as well as multiplication and addition operations.
- High speed reciprocal unit using piece-wise polynomial and binomial expansion is used.

Proposed SRUKF Architecture Design Methodology – Block Diagram



RESULTS AND DISCUSSION

RESULT AND DISCUSSION (1/2)

- Proposed SRUKF SLAM algorithm is implemented in MATLAB and compared with EKF SLAM and UKF SLAM.
- Comparison results show that SRUKF outperforms EKF and UKF SLAM as it remains highly stable during localization.
- SRUKF estimates new landmarks with greater accuracy as shown by the **Mean Absolute Error**.
  - MAE is the average error for the observed landmarks from true landmark position.
- SRUKF SLAM demonstrates 78% better accuracy compared to EKF SLAM and comparable accuracy of 10% from UKF SLAM.

TABLE I : MEAN ABSOLUTE ERROR FOR LANDMARK INITIALIZATION

State Estimator Methodology	Estimator Mean Absolute Error
EKF SLAM	0.526
UKF SLAM	0.1275
SRUKF SLAM	0.1143

RESULT AND DISCUSSION (2/2)

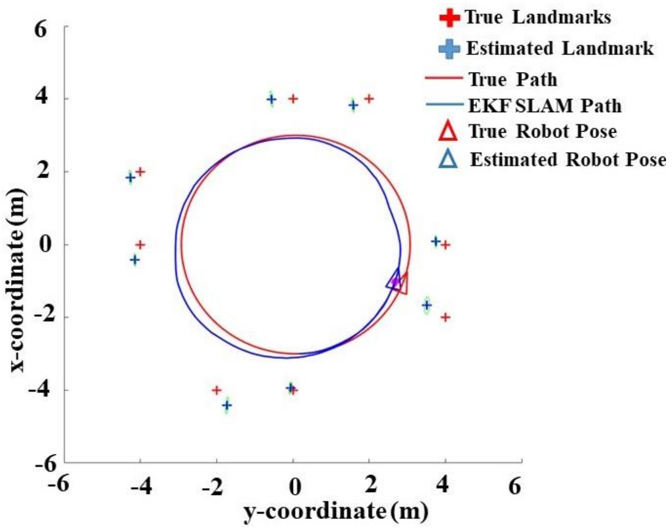
- Proposed architecture design for core computation unit was developed for three landmarks.
- Design was coded in Verilog and synthesized in Xilinx's Vivado 2018.2.
- Results show that the proposed methodology reduces hardware resource utilization and saves 91% of power hungry DSP cores.
- Timing analysis is done with maximum synthesizable frequency of 100 MHz.
- It demonstrates 20% faster design compared to state-of-the-art design.
- Overall power consumption reduced by 64% when compared to the state-of-the-art architecture core.

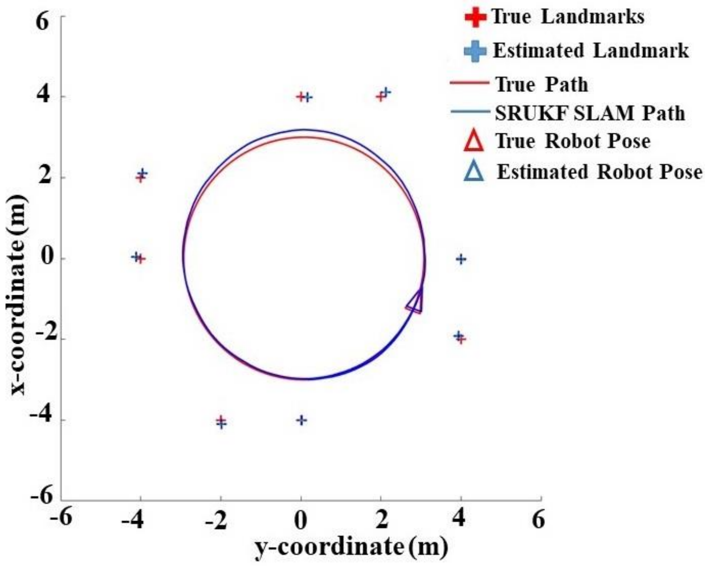
TABLE II: SYNTHESIS RESULTS

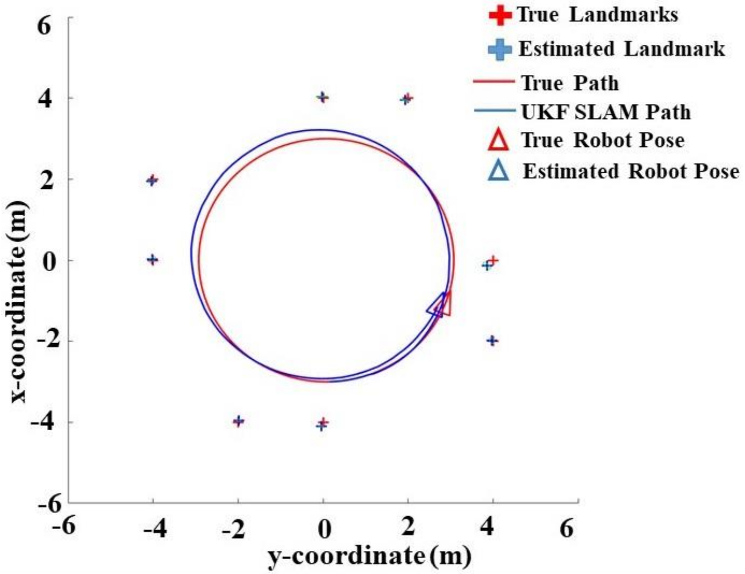
Resources	UKF Core	Proposed SRUKF Core	% Savings
LUT	8307	4813	42%
FF	6299	8215	-31% (overhead)
BRAM	16	12	25%
DSP48	35	3	91.4%
Timing	246 microseconds	195 microseconds	20%
Power	355 mW	127mW	64%

भारतीय प्रौद्योगिकी संस्थान हैदराबाद
Indian Institute of Technology Hyderabad

MATLAB and FPGA results comparison







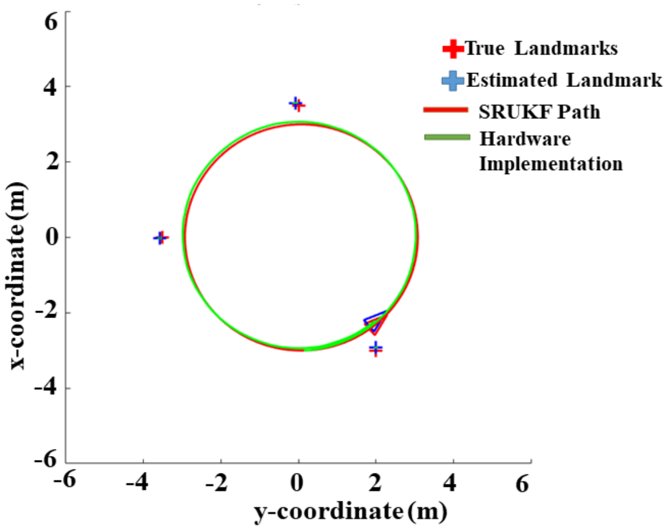


Fig 2: MATLAB Results for Robot path localization and landmark mapping using (a) EKF SLAM (b) UKF SLAM (c) SRUKF SLAM (d)SRUKF SLAM comparison between MATLAB and FPGA implementation

CONCLUSION

- The paper proposes SRUKF SLAM algorithm and its low complexity architecture design methodology using Householder CORDIC.
- The design is optimized for high speed and power efficient hardware implementation.
- Proposed methodology has comparable accuracy with UKF SLAM, demonstrates to have 78% higher accuracy compared to EKF SLAM.
- The speed of execution is improved by 20%, power hungry DSP modules by 91%, total power consumption is reduced 64% compared with the state-of-the-art architecture.
- Hardware software co-design methodology reduces the developmental time and enhances portability across multiple applications.

ACKNOWLEDGEMENT

CAD Tools are supported under MeitY SMDP-C2S Program, Government of India.

Indian Institute of Technology Hyderabad

REFERENCES (1/2)

1. J. Crassidis, F. Markley, and Y. Cheng, “Survey of nonlinear attitude estimation methods,” J. Guid. Control Dyn., vol. 30, no. 1, pp. 12–28, Jan./Feb. 2007.
2. H. Durrant-Whyte and T. Bailey, “Simultaneous localization and mapping: part I,” in IEEE Robotics Automation Magazine, vol. 13, no. 2, pp. 99-110, June 2006.
3. T. Bailey and H. Durrant-Whyte, “Simultaneous localization and mapping (SLAM): part II,” in IEEE Robotics Automation Magazine, vol. 13, no. 3, pp. 108-117, Sept. 2006.
4. S. Julier, J. Uhlmann, and H. F. Durrant-Whyte, “A new method for the nonlinear transformation of means and covariances in filters and estimators,” IEEE Trans. Automat. Control, vol. 45, no. 3, pp. 477–482, Mar. 2000.
5. Z. Kurt-Yavuz and S. Yavuz, “A comparison of EKF, UKF, FastSLAM2.0, and UKF-based FastSLAM algorithms,” 2012 IEEE 16th International Conference on Intelligent Engineering Systems (INES), Lisbon, 2012, pp. 37-43.
6. P. Kaminski, A. Bryson and S. Schmidt, “Discrete square root filtering: A survey of current techniques,” in IEEE Transactions on Automatic Control, vol. 16, no. 6, pp. 727-736, December 1971.
7. R. Merwe, E. Wan, “The square-root unscented Kalman filter for state and parameter-estimation”, Proc. IEEE Int. Conf. Acoustics Speech and Signal Processing (ICASSP), vol. 6, pp. 3461-3464, 2001.
8. S. Holmes, G. Klein, and D. W. Murray, “An O(N2) square root unscented Kalman filter for visual simultaneous localization and mapping,” IEEE Trans. Pattern Anal. Mach. Intell., vol. 31, no. 7, pp. 1251–1263, Jul. 2009.

भारतीय प्रौद्योगिकी संस्थान हैदराबाद
Indian Institute of Technology Hyderabad

REFERENCES (2/2)

9. S. Li and P. Ni, “Square-root unscented Kalman filter based simultaneous localization and mapping,” The 2010 IEEE International Conference on Information and Automation, Harbin, 2010, pp. 2384-2388.
10. J. Soh and X. Wu, “An FPGA-Based Unscented Kalman Filter for System-On-Chip Applications,” in IEEE Transactions on Circuits and Systems II: Express Briefs, vol. 64, no. 4, pp. 447-451, April 2017.
11. I. H. Kurniawan, J. Yoon and J. Park, “Multidimensional Householder based high-speed QR decomposition architecture for MIMO receivers,” 2013 IEEE International Symposium on Circuits and Systems (ISCAS), Beijing, 2013, pp. 2159-2162.
12. J. M. Delosme, “CORDIC algorithms: Theory and extensions”, Advanced Algorithms and Architectures for Signal Processing IV Proc. SPIE, vol. 1152, pp. 131-145, Aug. 1989.
13. A.W. Bojanczyk, A.O. Steinhardt, “Stability analysis of a Householderbased algorithm for downdating the Cholesky factorization”, SIAM J. Sci. Statist. Comput., 12 (No. 5), 1991.
14. J. Sola, T. Vidal-Calleja, J. Civera, J. M. M. Montiel, “Impact of landmark parametrization on monocular EKF-SLAM with points and lines”, International Journal of Computer Vision, vol. 97, pp. 339-368, 2012.

Thank You!